

Schutz 6.21

$$\text{why } [\nabla_\alpha, \nabla_\beta] F_\nu^M = R_{\alpha\beta}^M F_\nu^{\sigma} + R_{\nu\alpha\beta}^{\sigma} F_{\sigma}^M \quad (6.78)$$

have same sign for the two indices yet

$$V_{i\beta}^{\alpha} = V_{i\beta}^{\alpha} + T_{\mu\beta}^{\alpha} V^{\mu}, \quad V_{\alpha i\beta} = V_{\alpha i\beta} - T_{\alpha\beta}^{\mu} V_{\mu}$$

have opposite signs?

The difference in sign in $V_{i\beta}^{\alpha}$, $V_{\alpha i\beta}$ came from the fact that basis vectors and 1-forms are covariant and contravariant by definition, thus transforms oppositely with respect to the same direction.

But F_{ν}^M has both vector and 1-form dimension, thus its minus signs and plus signs for the bases are balanced.